Properties of White Dwarfs

Student Guide • Draft

Engage

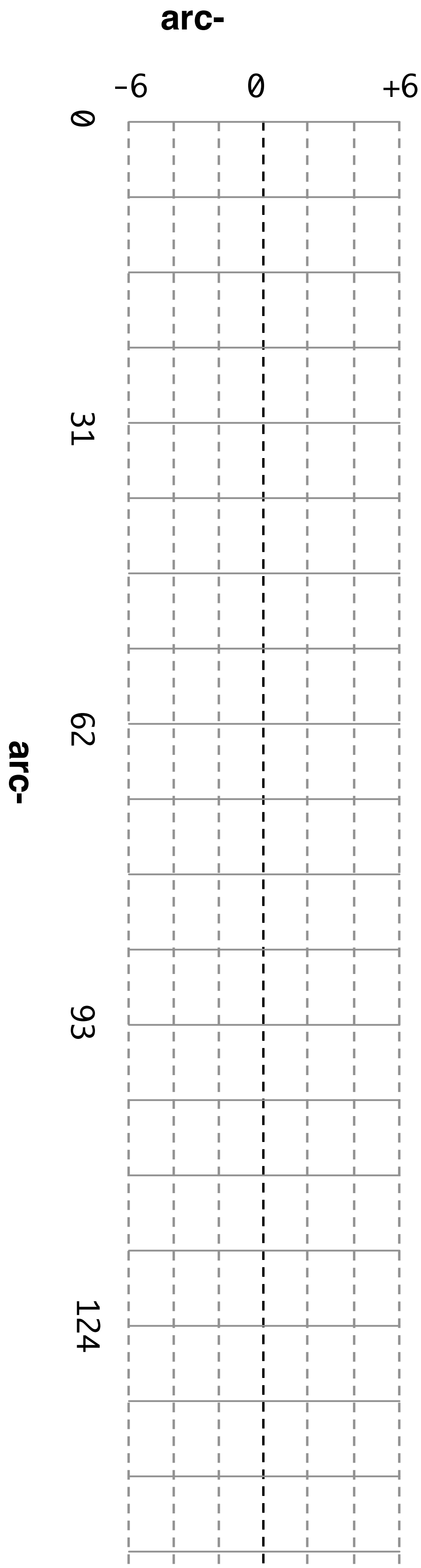
Sirius Plotting

The PowerPoint shows the motion of Sirius over the course of 100 years in five year "snapshots". As you view each frame, plot the position of Sirius on your graph, and label it with the date of the observation. Based on your plot of Sirius's position over time, answer the following questions:

A. What kind of motion does Sirius show? Justify your answer.

B. What variables can you identify to describe this motion?

C. What do you think is causing Sirius's motion?



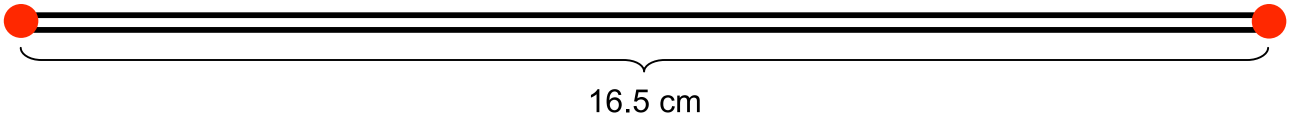
Sirius Plotting Graph

Explore Part I: Calculating the Mass of Sirius B

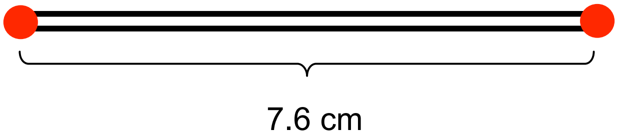
1. Make two string loops.

Sirius B loop: 33 centimeters	Sirius A loop: 15 centimeters
1. On a piece of cardboard, press a push pin into the cardboard.	1. On a piece of cardboard, press a push pin into the cardboard.
2. Measure **16.5 centimeters** away from the push pin, and mark the spot.	2. Measure **7.6 centimeters** away from the push pin, and mark the spot.
3. Insert another push pin into that spot.	3. Insert another push pin into that spot.
4. Make a **tight** string loop around the push pins, and tie off the string to complete the loop.	4. Make a **tight** string loop around the push pins, and tie off the string to complete the loop.
5. Check that the total length of your loop is 33 centimeters.	5. Check that the total length of your loop is 15 centimeters

Making the Sirius B loop (to scale – you can use this as a template)

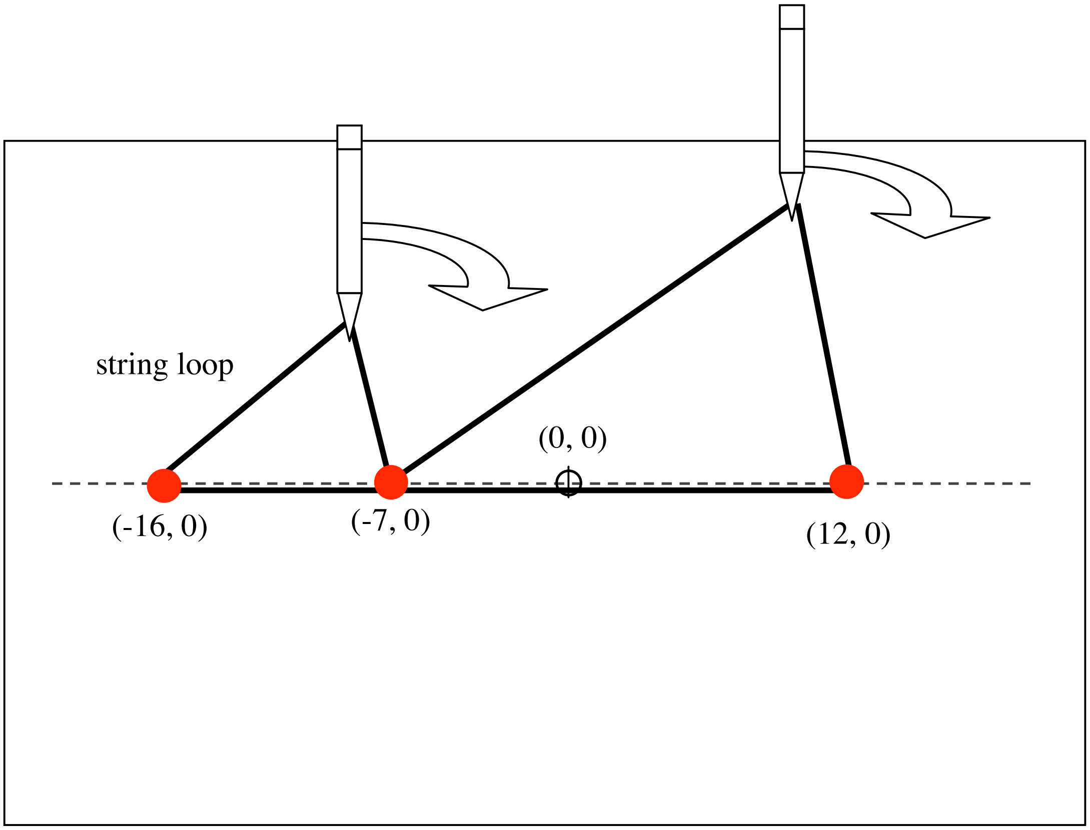


Making the Sirius A loop (to scale – you can use this as a template)



2. Set up the graph paper

Turn the graph paper lengthwise. Find the center of the graph paper to the nearest grid cross. Mark the center, and label it (0, 0) as an origin. Mark these points: (-7, 0), (-16, 0) and (12, 0).



Place the graph paper on top of the cardboard sheet. At each of these points (-16, 0), (-7, 0), and (12, 0) place a push pin.

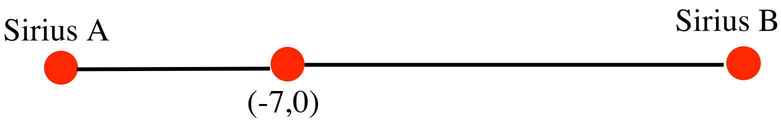
3. Draw the orbits of Sirius A and B:

Wrap the short loop around both push pins at (-16, 0) and (-7, 0). Let the loop guide your pencil as you draw a curve. This represents the orbit of Sirius A, the brightest star in the sky.
For the orbit of Sirius B, wrap the long loop around the push pins at (-7, 0) and (12, 0) and draw the curve with the loop guiding your pencil as you draw.

4. What is the mass of Sirius B?

Pick a point on the orbit of Sirius A. Put a push pin at that point. Put another push pin at (-7,0). With a ruler, line up the two push pins. Place the third push pin where the ruler crosses the orbit of Sirius B, on the *other side* of the push pin at (-7, 0).

Draw a line segment that connects the push pins.



Measure the distance between the center of mass (middle push pin) and each push pin. This is the distance between Sirius A or Sirius B and the barycenter of their obits.

Make a two column table:

Sirius A distance	Sirius B distance

Record your first set of data points, then repeat for six more points. Pick positions for Sirius A evenly around its orbit. The center of mass push pin (-7, 0) should stay fixed.

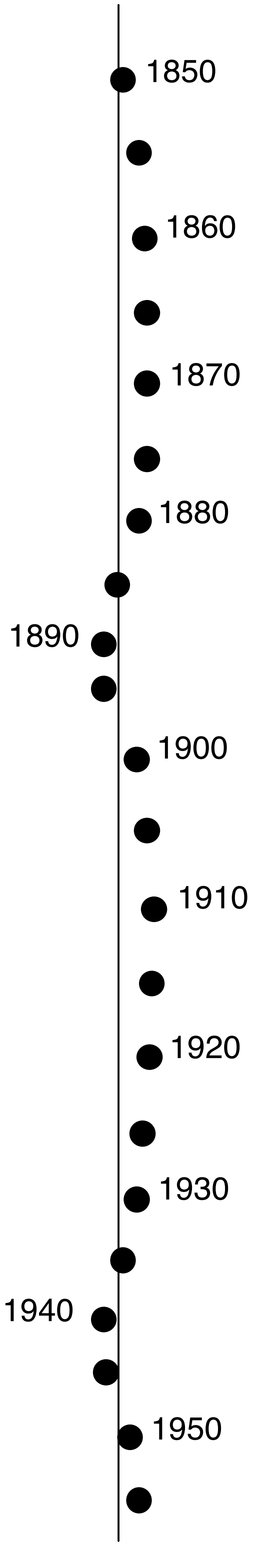
Plot your data on a graph Sirius B vs. Sirius A (**y** verses **x**) on the graph on the next page.

What kind of relationship does the data show between the two distances?

Note: the graph is 60 x 60 squares (try making 3 squares equal 1 cm)

History: Freidrick Whilhelm Bessel

Freidrick Bessel observed the motion of Sirius with respect to the background stars between 1831 and 1844. In that time, he saw part of this pattern in Sirius's motion. The dark spots represent Sirius A, the brightest star in the sky. Sirius is bright not only because it is intrinsically bright (about 2.5 times the luminosity of our Sun) but also because it is very close to us (only about 9 light years away).

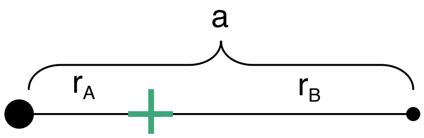


Based on your previous experiment, you know that Sirius has a companion. But where is it? Bessel did not see it. It was too faint and too close to Sirius. But Bessel had enough information to predict where to look.

Kepler's Law:

$$P^2 = \frac{4\pi^2 a^3}{G(m_A + m_B)}$$

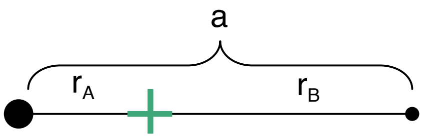
P: orbital period in seconds
a: semimajor axis in meters ($r_A + r_B$)
G: gravitational constant 6.67 x 10^{-11} N•m^2/kg^2
m: mass in kilograms.

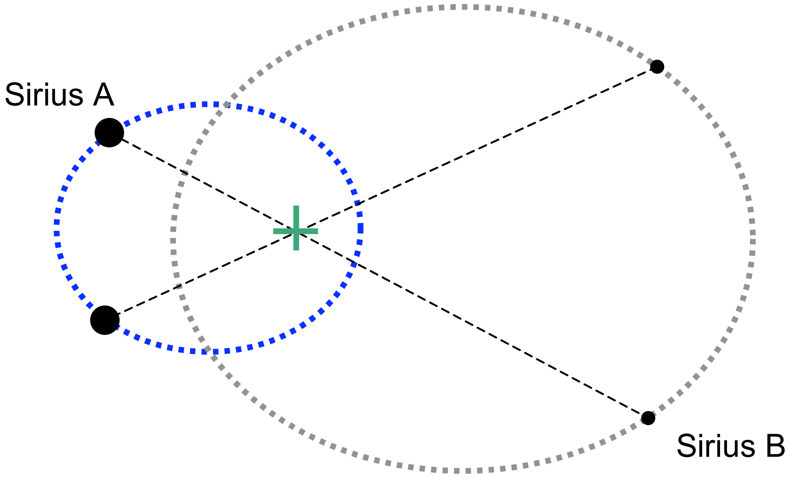


A simpler way to express this equation is to use units of the mass in terms of the Sun (solar mass), time in years, and distances in Astronomical Units (the semimajor axis of the Earth's orbit).

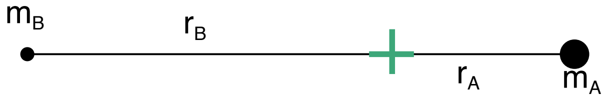
$$P^2 = \frac{a^3}{(m_A + m_B)}$$

P: orbital period in years
a: semimajor axis in Astronomical Units
m: ratio of the star's mass to the sun's mass (solar masses)





For the Sirius binary system:



If a = 20.21 AU and P = 50 years
What is the total mass? ($m_A + m_B$)?

Calculating the mass of Sirius B

Now that you know the sum of the masses in the binary star system, you can calculate the mass of Sirius B. Applying conservation of angular momentum, the product of the mass and orbital radius for Sirius A and B must balance.

$$m_A r_A = m_B r_B$$

$$\frac{r_A}{r_B} = \frac{m_B}{m_A}$$

(mass of Sirius A = 2.2 solar masses)

From your earlier plot, you know the ratio of the distances so you also know the ratio of the masses. Combining the mass ratio with the sum of the masses (in solar mass units), you can calculate the mass of Sirius B.

Equation 1 $$m_A = m_B \frac{r_B}{r_A}$$

Equation 2 $$m_A + m_B = 3.3$$

Isolate m_B and calculate the mass for Sirius B.

Size of a White Dwarf

The name dwarf indicates that this may be a small object. If you know the white dwarf's temperature and luminosity (amount of energy it radiates per second), then you can calculate its radius.

$$L = 4\pi R^2 \times \sigma T^4$$

L: luminosity in watts (W)
T: temperature of the surface in Kelvin (K)
R: radius of the white dwarf in meters
σ: Stephan-Boltzmann constant = 5.67×10^{-8} ($\mathrm{W/[m^2K^4]}$)
π: pi, the ratio of a circle's circumference to its diameter

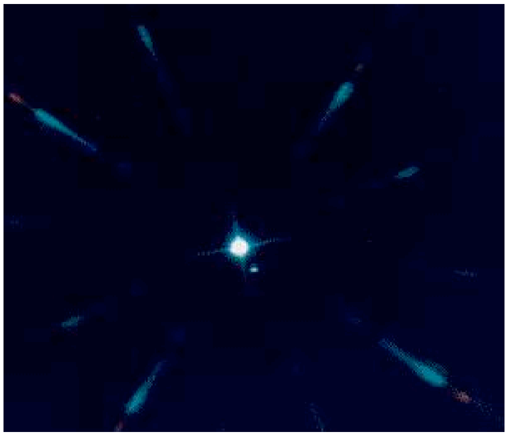
Astronomical Magnitudes

Astronomers measure brightness in units of magnitudes: the fainter the object, the larger the magnitude. For instance, our Sun's apparent magnitude is –26, while the faintest stars you can see in the sky without optical aid is +6. For a difference of 5 magnitudes, the flux changes by a factor of 100. But to your eye and brain, an object 1 magnitude lower than another looks about twice as bright, instead of $\sqrt[5]{100} = 2.5$ times as bright.

A spectroscopic analysis of Sirius B shows that the surface temperature is about 27,000 Kelvin. Compare that to our Sun at 5,770 Kelvin, and you understand why these objects when first discovered were called "white". They are very hot. But what about "dwarf"? Based on Bessel's calculations for Sirius's companion, astronomers were able to find Sirius B and measure its apparent brightness (magnitude), which turned out to be very dim: +8 compared to Sirius at –1.4. That difference in brightness magnitudes means that Sirius A radiates 5,864 times more light than Sirius B in the *visible region* of the electromagnetic spectrum.

Reflect

Before you calculate Sirius B's luminosity, think about what the astronomers had discovered. Sirius A and B orbit a common center of mass (barycenter) with "A" more luminous than our Sun, and "B" thousands of times fainter than "A". Sirius B is so faint that it barely shows up in a photograph. Sirius B certainly was an odd object, unlike any star previously known at the time. It was about the same mass as our Sun, yet extremely dim. In the picture, Sirius B is the tiny speck next to Sirius A. In order to reveal Sirius B, Sirius A is overexposed so the size difference in this picture is not real.



McDonald Observatory photo

Calculate the diameter of Sirius B

Since they also knew the distance to Sirius B, astronomers calculated Sirius B's luminosity. For L=1.15 x 10^{25} Watts and T=27,000 K, calculate the radius of the star in meters. For comparison, the Sun is 1.4 x 10^{9} meters in diameter and the Earth is about 1.28 x 10^{7} meters in diameter. Express Sirius B's diameter as a ratio: Sirius B's radius : Sun's radius and Sirius B's radius **:** Earth's radius.

Density

Based on your calculation for Sirius B's radius, **<u>calculate its average density.</u>** The volume of a sphere is:

$$V = \frac{4}{3}\pi R^3$$

Density is the total mass divided by the volume.

(Note: the Sun's mass is 2.0 x 10^{30} kg)

For comparison, water is 1 gram per cubic centimeter, or 10^{3} kg per cubic meter. The Sun's average density is about 1.4 grams per cubic centimeter, or 1.4 x 10^{3} kg per cubic meter.

Explain

1. How does Sirius B compare to the Sun in terms of:

Property	**Sun**	**Sirius B**	**Sirius B / Sun**
Luminosity	*1 (3.26 x 10^{26} Watts)*		
Diameter			
Average Density			

2. Imagine that you are an astronomer in the early 1900's, and were just learning about the strange properties of Sirius B. Why do you think your fellow astronomers might have difficulty accepting the proposed properties of Sirius B?

3. Summarize the properties of a white dwarf in one sentence, as if you were making up a newspaper headline.

Elaborate

Since the early 20th century, many white dwarfs have been discovered. Since they are so small and dim, they are hard to find. But today at the University of Texas at Austin, a group of astronomers is not only looking for white dwarfs, they are trying to use them as clocks to determine the age of our Galaxy. Read the following StarDate scripts about white dwarfs.

Please note that these scripts are dated. Any references about when to view an object may be inaccurate. See the underlined sentences in the scripts.

Dog and Pup January 20, 2003

The dog star Sirius trots across the southern sky on winter nights. It's the brightest star in the entire night sky. Only the Moon and a couple of planets regularly outshine it. Tonight, Sirius rises in the southeast about the time darkness falls, and climbs into good view by an hour later. It stands highest in the sky after midnight.

Sirius appears so bright in part because it really IS a fairly bright star, but mostly because it's one of our closest neighbors -- less than nine light-years away.

That closeness made Sirius a favorite target for astronomers who were building new telescopes and new techniques in the 19th century. In 1844, for example, German astronomer Friedrich Bessel discovered that something was tugging Sirius back and forth a little bit -- an unseen star. Alvan Clark first saw this star in 1862. It's small and faint, so it's hard to find beside Sirius. Since it's a little companion to the DOG star, astronomers called it the Pup.

A half-century later, astronomers found that the Pup was unlike any other star they'd ever seen. Its surface is extremely hot, yet the star produces so little light that it must be tiny -- no bigger than Earth. But it's about as massive as the Sun, which has a diameter a hundred times GREATER than Earth's.

The Pup was the first of a previously unseen type of star called a white dwarf -- the crushed corpse of a once normal star.

White Dwarfs January 21, 2003

As our Sun ages, it'll undergo dramatic changes. It'll get hotter and brighter in the core, then puff up like a balloon. Eventually, it'll shed its outer layers, surrounding itself with a colorful but ephemeral bubble of gas. As the gas dissipates, only the Sun's collapsed core will remain -- a cosmic cinder slowly fading from sight.

This dead remnant is called a white dwarf. Billions of white dwarfs probably inhabit the Milky Way galaxy. But they're so faint that they're hard to see. In fact, not a single white dwarf is visible to the unaided eye. The closest one is a companion to Sirius, the brightest star in the night sky, which wheels across the south on winter nights. It's less than nine light-years away. But the white dwarf is so faint compared to Sirius that it wasn't discovered until the 1860s.

A white dwarf is the inevitable fate of a certain class of stars -- stars that are no more than about 8 times as massive as the Sun. As these stars age, they "burn up" the hydrogen fuel in their cores. They then burn heavier elements, until they can't generate the temperatures needed to keep going. Their cores collapse, forming ultradense, hot balls about as big as Earth. Over many billions of years, they'll cool and fade from sight.

Evolution to a white dwarf is a long-term project, though. The Sun won't reach that final stage of life for another five or six billion years.

Setting a Date July 17, 2002

After decades of debate, astronomers are reaching a rough consensus on the age of the Universe. And recent observations by Hubble Space Telescope not only strengthen this estimate, they may help astronomers narrow the range even more.

Several years ago, astronomers used the orbiting telescope to measure the distances to remote galaxies. These measurements reveal how fast the universe is expanding now, and how the rate of expansion has changed over time. From these measurements, the astronomers estimated that the Big Bang took place 13 billion to 14 billion years ago.

More recently, astronomers used Hubble to measure the ages of the oldest stars yet discovered -- DEAD stars called white dwarfs. These stars no longer produce energy in their cores. They shine simply by radiating their intense heat into space. The older a white dwarf, the cooler its surface, so measuring its temperature provides a good estimate of its age.

Astronomers used Hubble to study white dwarfs in a star cluster called M4. The telescope watched M4 for several days, and found some of the coolest white dwarfs ever discovered -- and therefore, some of the oldest. From the temperatures of these stars, astronomers estimate that they're 12 billion to 13 billion years old. Since the stars probably formed soon after the Big Bang, their ages agree well with the earlier estimates of the age of the Universe.