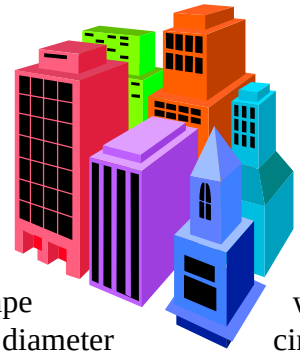
Prince William Counties Math and Science Departments Present:

THE PENTAGON LAB



INTRODUCTION

Geometric shapes are all around us. Geometry refers to the measuring and comparing of shapes. Using simple steps that were taught since 5th grade (and one advanced step) you will calculate the largest equilateral pentagon (a five sided shape with all five sides being exactly the same length) that will fit into a fixed diameter circle. When finished you will draw the pentagon and using the law of reflection (see *the Law of Angles Lab*) you will use a laser to test your calculations and measurements; Why lasers? Because they're FUN!

You will be sketching simple shapes and using math to determine the relationships. USE METRICS!

Table Setup

1. Tape a piece of construction paper in the center of your table.

2. Using a compass, draw a **CIRCLE** in the center of the paper large enough to draw other lines in later. Record the **RADIUS** of the circle here: r = _____ cm. You need to be exact in your measurements!

3. On a separate sheet of paper sketch (not with a compass) a circle similar in size.

Circle and Pentagon Identification

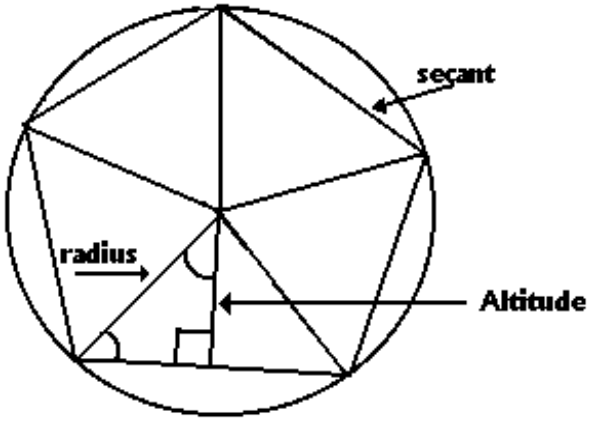
1. Sketch a five-sided figure inside the circle you just drew so that the lines meet at the edge of the circle. The sides should create a **PENTAGON** inside the circle.

1. From the sketch, place a point roughly in the center of the circle.

2. Draw 5 lines from that point out to the circle where any two lines meet. In a perfect circle, these five lines would all be the same in length. They would all be the **RADII**. Connect lines between where each of the radii meet the edge of the circle creating a pentagon inside the circle. The sides of the pentagon are the interior segments of **SECANTS**, or a **CHORD**

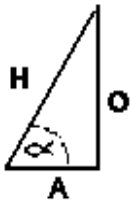
3. Assuming the lines are all evenly spaced out (your pentagon was drawn correctly) the circle becomes divided into _____ parts. The whole circle is made up of _____ degrees so each **INTERIOR ANGLE** would be _____ degrees.

4. Dropping an **ALTITUDE** from the center of the circle to the midpoint of one side of the pentagon in the circle produces a line that **BISECTS** both the central angle and the secant and is **PERPENDICULAR** to the secant.



5. The central angle was 72 degrees so when it is bisected it produces two new angles of: _____ degrees. The third interior angle of the triangle is: _____ degrees, because all angles in a triangle must add up to _____.

6. Using a **TRIGONOMETRY** (this was the step that some of you may not have had…Trig is basically looking at how does one part of a figure compare to another) function called Cosine we will determine the length of the bisected chord. Cosine simply compares the length of the line "along side" (adjacent) the angle you are referring to with the length of the longest side in a right triangle.



Cos α = Adjacent/Hypotenuse, or:

Cos 54 degrees = <u>Bisected Length of the chord</u>
Length of the Radius of the circle

Algebraically manipulating this formula produces:

(Cos36*)(Length of the Radius) = (Bisected length of the chord)

7. We now have the bisected (divided in ½) length of the chord so the total length of the chord, or one side of the pentagon, should be: _____cm.

CHECKING MATH and LASERS!

1. Return to the original circle you drew. Draw a small line somewhere along the edge of the circle. From that point use a ruler and measure out EXACTLY the length of the chord you just calculated above. Place a small line along the edge at that point. Continue until you have a total of FIVE points.

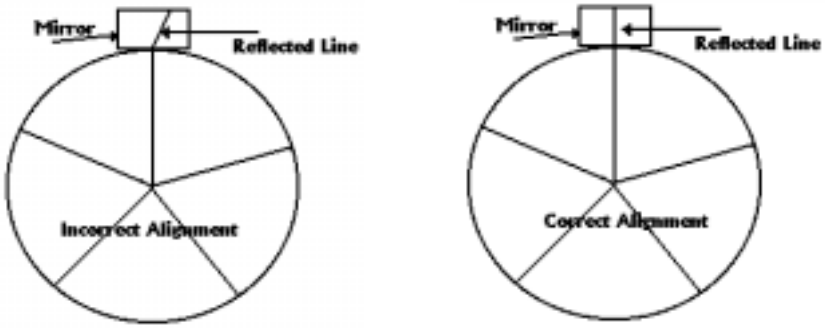
2. Pick ONLY TWO consecutive points and connect them.

3. From each point use a ruler and draw a line back to the center of the circle.

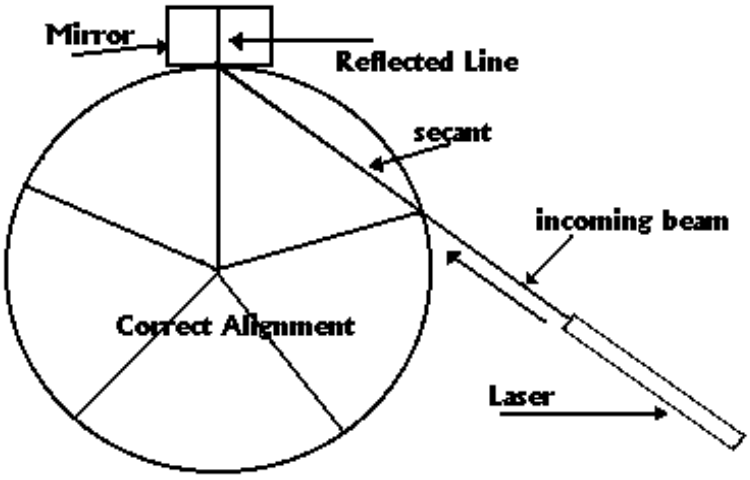
4. Using FOUR mirrors complete the following alignment step for each mirror:

A. Tape the mirror horizontally to edge of a book and place it along the edge of the circle so that the lines you drew are in the middle of each mirror.

B. Align the mirror having one partner look straight down the line into the mirror and one partner adjust the mirror accordingly. (The line should go straight into the mirror...this makes sense when you do it)



5. Taking a LASER (and PLEASE BE CAREFUL WITH THE LASERS!) Shine the beam over the mark on the circle that has no mirror. Be careful to have the beam cross the mark in the direction of the first mirror aligned,over the one chord you drew, and have it hit the mirror on the reflected line. This also makes sense when you do it.



Add fog or aerosol to the air around your circle to determine how accurate your calculations, measurements and alignments were. Measure how far away from the starting point your reflected beam was.

MATH QUESTIONS:

1. Define these key terms:
    a. Geometry
    b. Equilateral
    c. Pentagon
    d. Radius
    e. Interior Angle
    f. Chord
    g. Secant
    h. Bisect
    i. Trigonometry
    j. Altitude

2. What type of triangle was produced when the Altitude was drawn to the secant?

3. What are the exterior angles made by the secants in the Pentagon?

4. What does the theorem say that allows you to make statements about the altitude drawn to the secant?

SCIENCE QUESTIONS:

1. Define these key terms:
    a. Laser
    b. Reflection
    c. "Law of Reflection"
    d. Incident beam

2. The principle of Reflection allows the beams to be bounced around the pentagon. How is it that (what science principle) we know what the angles they produce are?

3. When a wave is reflected back it Interferes with other waves being sent out. What are the two types of wave interference? What does each type do?

4. When a reflected wave interferes with a new wave the interference patterns produce combinations of new waves. This is known as the _____ effect.

5. Using the speed of light, $c = 3.0 \times 10^8$m/s (roughly 186,000 miles per second) and the formula $\mathbf{v = d/t}$ we can substitute c for v and get: $\mathbf{c = d/t}$. We can manipulate the formula to isolate t giving the formula: $\mathbf{t = d/c}$.

    Knowing the speed of light, $c = 3.0 \times 10^8$m/s
    and given the distance traveled, d, is 5 times the length of one secant: _____ (convert to METERS!),

    Substitute into the formula for time and determine how long it took the light wave to travel around your pentagon. t= ____seconds.